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EFFECTS OF AXIAL HEAT CONDUCTION IN A VERTICAL FLAT PLATE ON FREE CONVECTION HEAT TRANSFER

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Abstract – Steady two-dimensional conjugate heat-transfer problems of free convection from a vertical flat plate have been analyzed mainly by the method of using local similarity solution of free convection boundary layer. The two thermal boundary conditions considered here are constant temperature and constant heat flux at the outside surface of the flat plate. The effects of axial conduction in the flat plate on the interfacial temperature are significant in the constant heat flux case, and correlated by the dimensionless parameter KD. Comparisons with the results of the finite difference method and related experiments indicated the appropriateness of the present analytical solutions.

NOMENCLATURE

a, thermal diffusivity of a fluid;
$a_0 \ldots a_4$, unknown coefficients in equation (21);
$A(Pr)$, $B(Pr)$, constants in equations (27) and (28);
C, constant defined by equation (16);
d, thickness of plate;
D, d^*, dimensionless form of d, $= Gd$, $= d/l$;
F, dimensionless interfacial temperature defined by equation (10);
g, acceleration due to gravity;
G, constant defined by equations (11) and (12);
Grx, Grl, Grashof number,
$$\frac{g\beta(T_w - T_\infty)x^3}{\nu^2}, = \frac{g\beta(T_O - T_\infty)l^3}{\nu^2};$$
Grx^*, Grl^*, modified Grashof number,
$$= \frac{g\beta q_w x^4}{\nu^2 k_s}, = \frac{g\beta q_O l^4}{\nu^2 k_f};$$
h, hm; local and average heat-transfer coefficients;
k_s, k_f; thermal conductivities of a solid and a fluid;
K, ratio of thermal conductivity, $= k_s/k_f$;
l, height of flat plate;
L, dimensionless height, $= Gl$;
Nux, $\overline{Nux}$, local and average Nusselt number, $= hx/k_f$, $= hmx/k_f$;
Pr, Prandtl number;
q_0, q_w, heat flux at outside surface and at interface;
Q_0, Q_w, dimensionless heat flux, $= lq_0/(k_s\Delta T)$, $= lq_w/(k_s\Delta T)$;
T_s, T_f, temperature of a solid and a fluid;
T_w, T_0, temperature at interface and temperature given in advance at outside surface;
u, v, velocity in x and y directions;
x, vertical distance from leading edge;
X, x^*, dimensionless form of x, $= Gx$, $= x/l$;
y, distance normal to vertical surface of plate;
Y, y^*, dimensionless forms of y, $= Gy$, $= y/l$;
z, horizontal distance;
β, volume coefficient of thermal expansion;
ΔT, temperature difference defined by equations (11) and (12);
ε, emittance of heated plate;
η, ξ, dimensionless variables defined by equation (14);
ν, kinematic viscosity.

Subscripts

0, value of uniform temperature and uniform heat flux given in advance;
w, value at interface;
f, fluid;
s, solid;
$*$, dimensionless length based on l, except for Grx^* and Grl^*.

1. INTRODUCTION

IT IS well known that, when convective heat-transfer results are strongly dependent on the thermal boundary condition, consideration of convective heat-transfer problems as conjugated problems is necessary to obtain physically more strict results. A good many research efforts, both experimental and theoretical, has been devoted to the conjugate problems of forced convection heat transfer. But a few publications have

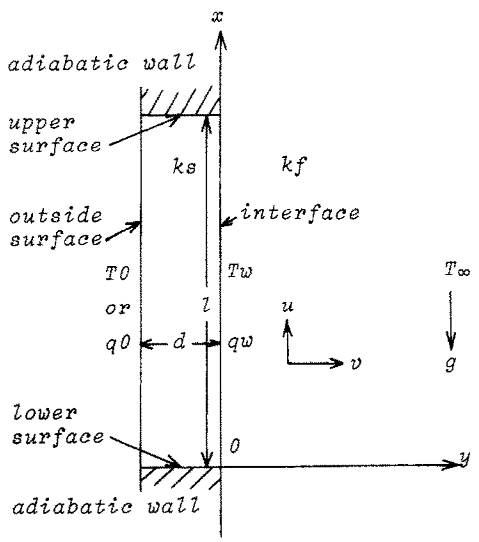

FIG. 1. A vertical flat plate and coordinate system.

been devoted to the conjugate problems of free convection.

Gdalevich and Fertman [1] discussed the method, the specifics and the principal results in the previously obtained solutions of conjugate problems of free convection. They stated conclusively that the use of numerical methods for solving the initial system of governing partial differential equations, such as finite difference method, is evidently the most promising in studies of conjugate free convection. As Kelleher and Yang [2] pointed out, the analytical treatments, used extensively in conjugate forced convection problems [3], are difficult due to matching a non-linear solution of free convection in a fluid with a linear conduction solution in a solid body at the solid–fluid interface.

In the problems of conjugate free convection about a tapered, downward projecting fin of a simple power law form, successful analytical solutions could be obtained by Lock and Gunn [4].

Generally, it seems difficult to obtain more exact solutions of conjugate free convection by the analytical treatment than by the numerical method such as finite difference method [5].

Nevertheless, analytical solutions of conjugate free convection, if it can be obtained, may be useful to seize the main features of the conjugate problem and to find the dimensionless parameter which controls the characteristics of the conjugate free convection. Chida and Katto [6] performed studies of conjugate problems in this direction by the use of vectorial dimensional analysis. They applied their method to the interpretation of previously studied conjugate heat transfer problems.

In this work, the vertical flat plate which has a height l, a thickness d and a constant conductivity k_s is heated from the outside surface by an external source (a condensing vapor, electrical source, etc.), as is shown in Figure 1. Heat moves through it by two-dimensional conduction and is transferred from the solid–fluid interface by laminar free convection to a fluid. Over and under the heated flat plate, two semi-infinite flat plates with the same thickness and zero conductivity, are placed vertically without any gap. Two thermal boundary conditions prescribed at the outside surface will be considered here. These are the constant temperature case (here after this is called case 1) and the constant heat flux case (case 2), which are realized in many practical applications and experiments.

It is the purpose of this paper to predict theoretically the temperature and the heat-transfer rate at the solid–fluid interface, which are determined by the common solution of energy equations for the fluid and the vertical flat plate. It is important to show the analytical method which can be applied to other various boundary conditions, and to elucidate quantitatively what dimensionless parameters play the most important role in each case of the conjugate problem.

The local similarity solution to the boundary layer equations of free convection from a vertical surface with an arbitrary temperature distribution, and Fourier's series solution to the steady two-dimensional heat conduction equation for the flat plate are matched, interfacially, satisfying the continuity of temperature and heat flux.

In order to show the appropriateness of the present analytical solution, numerical results of the interfacial temperature are compared with the numerical solution using the finite difference method and related experiments.

2. BASIC EQUATIONS AND SOLUTIONS

The physical model and coordinate system are shown in Fig. 1. Steady two-dimensional free convection from a vertical solid wall is given by the usual laminar boundary layer equations:

$$0 \leqq y \leqq \infty, \quad -\infty \leqq x \leqq \infty:$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T_f - T_\infty) + v\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T_f}{\partial x} + v\frac{\partial T_f}{\partial y} = a\frac{\partial^2 T_f}{\partial y^2}. \tag{3}$$

Steady two-dimensional temperature fields for the flat plate are given by equation (4):

$$-d \leqq y \leqq 0, \quad 0 \leqq x \leqq l:$$

$$\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} = 0. \tag{4}$$

Equation (3) and (4) are coupled by continuity conditions at the solid–fluid interface:

$$0 \leqq x \leqq l:$$

$$T_s(x,0) = T_f(x,0) = T_w(x) \tag{5}$$

and

$$k_s \frac{\partial T_s}{\partial y}(x, -0) = k_f \frac{\partial T_f}{\partial y}(x, +0). \quad (6)$$

Numerous combinations of the boundary conditions at the outer surface, upper surface and lower surface of the cross-section of the vertical flat plate are of some interest, but to illustrate the method of analysis, the following boundary conditions will be considered in addition to the fluid–solid interfacial condition:

(i) $-d \leqq y \leqq 0$:

$$\frac{\partial T_s}{\partial x}(0, y) = \frac{\partial T_s}{\partial x}(l, y) = 0$$

(ii) $0 \leqq x \leqq l$:

$$T_s(x, -d) = T_0 = \text{const}$$

(constant temperature at the outside surface, case 1)

or

$$k_s \frac{\partial T_s}{\partial y}(x, -d) = q_0 = \text{const}$$

(constant heat flux at the outside surface, case 2)

(iii) $-\infty < x < \infty$:

$$T_f(x, \infty) = T_\infty = \text{const}$$

(iv) $0 \leqq x \leqq l$:

$$u(x, 0) = v(x, 0) = u(x, \infty) = v(x, \infty) = 0. \quad (7)$$

Hereafter, the following dimensionless variables are used:

$$X = Gx,\ Y = Gy,\ L = Gl,\ D = Gd, \quad (8)$$

$$x^* = \frac{x}{l} = \frac{X}{L},\ y^* = \frac{y}{l} = \frac{Y}{L},\ d^* = \frac{d}{l} = \frac{D}{L}, \quad (9)$$

$$F = \frac{T_w - T_\infty}{\Delta T}. \quad (10)$$

Where

$$\text{case 1: } G = \left(\frac{g\beta\Delta T}{\nu^2}\right)^{1/3},\quad \Delta T = T_0 - T_\infty \quad (11)$$

$$\text{case 2: } G = \left(\frac{g\beta q_0}{\nu^2 k_f}\right)^{1/4},\quad \Delta T = \left(\frac{g\beta k_f^3}{\nu^2 q_0^3}\right)^{-1/4}. \quad (12)$$

2.1. *Local similarity solution for laminar free convection*

Kao, Domoto and Elrod [7] have developed a technique for the solution of free convection problems on a vertical flat plate with arbitrary (though smooth) temperature. If the interfacial temperature $T_w(x)$ in equation (5) is assumed to be known, their technique can be applied to these problems. From the standpoint of local similarity, the first approximation solution of equations (1), (2) and (3) with boundary conditions (5), and (iii) and (iv) in equation (7) have been obtained and have given the dimensionless heat flux at the interface. The result given by Kao, Domoto and Elrod [7] for $Pr = 0.70$ can be closely approximated by the following equation:

$$Q_w = -CF^{3/2}\xi^{-1/4}[-0.4995 - 0.2710\eta]. \quad (13)$$

Where

$$\eta = \frac{\xi\, dF}{F^2 dx^*},\quad \xi = \int_0^{x^*} F dx^* \quad (14)$$

and constant C is given by:

$$\text{case 1: } C = \frac{1}{\sqrt{2}} Grl^{1/4} \quad (15)$$

$$\text{case 2: } C = \frac{1}{\sqrt{2}} Grl^{*3/16}. \quad (16)$$

The approximation given by equation (13) is very accurate for $0 \leqq \eta \leqq 1/6$. $\eta = 0$ and $\eta = 1/6$ correspond to the constant interfacial temperature and the constant interfacial heat flux, respectively.

2.2. *Analytical solution for a flat plate*

The temperature distribution in the flat plate can be obtained by solving equation (4) subject to boundary conditions (5) and (i), (ii) in equation (7). If $T_w(x)$ in equation (5) is assumed to be known, then the solution can be obtained by the classical methods discussed by Carslaw and Jaeger [8] to give the following dimensionless heat flux at the interface:

case 1:

$$Q_w = -\frac{K}{d^*}\int_0^1 (F-1)\,dx^* - 2K\sum_{n=1}^{\infty} n\pi \coth(n\pi d^*) \times \cos(n\pi x^*)\int_0^1 (F-1)\cos(n\pi x^*)\,dx^* \quad (17)$$

case 2:

$$Q_w = KQ_0 - 2K\sum_{n-1}^{\infty} n\pi \tanh(n\pi d^*)\cos(n\pi x^*) \times \int_0^1 F\cos(n\pi x^*)\,dx^*. \quad (18)$$

The leading terms of the right hand side of equations (17) and (18) contain K/d^* and Kd^*, respectively, by the following approximation:

$$\tanh x = x + O(x^3) \quad (x < \pi/2).$$

KQ_0 in equation (18) is excepted.

It can be seen that the dominant dimensionless parameters are KL/D for case 1 and KD/L for case 2.

2.3. *Conjugate problem solution*

By the continuity condition (6) for the heat flux at the interface, the right hand side of equation (13) can be equated to the right hand sides of equations (17) and (18). Then the non-linear integro-differential equations (19) and (20) for each case are obtained and can be solved to know the interfacial temperature:

case 1:

$$\frac{1}{\sqrt{2}} Grl^{1/4} F^{3/2} \xi^{-1/4} (0.4995 + 0.2710\,\eta)$$

$$= -\frac{K}{d^*}\int_0^1 (F-1)\,dx^* - 2K\sum_{n=1}^{\infty} n\pi \coth(n\pi d^*) \times \cos(n\pi x^*)\int_0^1 (F-1)\cos(n\pi x^*)\,dx^* \quad (19)$$

case 2:

$$\frac{1}{\sqrt{2}} Grl^{*3/16} F^{3/2} \xi^{-1/4} (0.4995 + 0.2710\,\eta)$$

$$= KQ_0 - 2K\sum_{n=1}^{\infty} n\pi \tanh(n\pi d^*)\cos(n\pi x^*) \times \int_0^1 F\cos(n\pi x^*)\,dx^*. \quad (20)$$

These equations seem to be too difficult to solve exactly. So we tried to obtain approximate solutions by a procedure outlined as follows (further details can be found in [9]:

In the first place, the unknown interfacial temperature distribution $F(x^*)$ is expressed in a simple polynomial equation including the unknown coefficients:

$$F(x^*) = a_0 + a_1x^* + a_2x^{*2} + a_3x^{*3} + a_4x^{*4} \quad (21)$$

Equation (21) is substituted for equations (19) and (20). Then the right hand sides of equations (19) and (20), that indicate convective terms, are expanded in the power series of x^*. Both sides of the obtained equations are multiplied by cos $(m\pi x^*)$ and integrated by x^* between 0 and 1 ($m = 1, 2, 3, \ldots$). By the orthogonality of the cosine function, the systems for the nonlinear simultaneous equations of a_n are obtained. These systems can be solved by the method of successive iteration, in which the nonlinear terms of a_n are supposed to be constant and then, the consequent systems of linear simultaneous equations of a_n can be solved easily.

After about 10 iterative cycles, all cases converged within the $10^{-3}\%$ changes of unknown a_n.

Heat-transfer coefficient is evaluated by the following equations:

$$q_w = h(T_w - T_\infty). \quad (22)$$

Then, the local Nusselt number is calculated by the following equations with resultant F.

case 1:

$$\frac{Nux}{Grx^{1/4}} = +\frac{1}{\sqrt{2}}(0.4995 + 0.2710\,\eta)\left(\frac{Fx^*}{\xi}\right)^{1/4} \quad (23)$$

case 2:

$$\frac{Nux}{Grx^{*1/5}} = \left[\frac{1}{\sqrt{2}}(0.4995 + 0.2710\,\eta) \cdot \left(\frac{F\cdot x^*}{\xi}\right)^{1/4}\right]^{4/5}. \quad (24)$$

Average Nusselt number is calculated by the following equations:

case 1:

$$\frac{\overline{Nul}}{Grl^{1/4}} = \frac{1}{\sqrt{2}}\int_0^1 \xi^{-1/4} F^{3/2} (0.4995 + 0.2710\,\eta)\,dx^* \quad (25)$$

case 2:

$$\frac{\overline{Nul}}{Grx^{*1/5}} = Grl^{*1/20}\int_0^1 F^{-1}\,dx^*. \quad (26)$$

3. FINITE-DIFFERENCE SOLUTIONS

For the semi-infinite heated flat plate ($l = \infty$), finite-difference solutions have been obtained. The details of the solutions are given in [10] and [11]. The outline of the procedures is as follows:

Governing partial differential equations of unsteady free convection in the fluid and unsteady heat conduction in the flat plate have been transformed into the explicit upwind finite-difference equations, including continuity conditions (5) and (6), and boundary conditions (7). Numerical calculations started just after a step change in temperature or heat flux at the outside surface. When the change of the dimensionless interfacial temperature after 100 time steps reached within 0.01%, the numerical solution was considered to have reached its steady state.

Particularly for case 2, a larger KD value necessitated a larger number of time steps. When KD is larger than 3000, the present finite-difference technique is not so efficient. Ziness [5] indicated the similar tendency with the exception of the effects of D. These difficulties may be removed by the introduction of the iterative cycles proposed by Gdalevich and Fertman [1]. But they did not indicate the numerical results.

4. INTERFACIAL TEMPERATURE OF THE FLAT PLATE WITHOUT AXIAL HEAT CONDUCTION

In order to examine the effects of axial heat conduction in the flat plate on the interfacial temperature, the following simplified treatment of the conjugate problems will be compared. The previous solutions of the free convection heat transfer gave the local heat-transfer coefficients at the solid–fluid interface which are given by the following equations:

(1) Constant interfacial temperature:

$$Nux = A(Pr)\cdot Grx^{1/4} \quad (27)$$

(2) Constant interfacial heat flux:

$$Nux = B(Pr)\cdot Grx^{*1/5} \quad (28)$$

When $Pr = 0.70$, $A(Pr) = 0.353$ and $B(Pr) = 0.483$, from equation (13).

It is assumed that local heat-transfer coefficients at the solid–fluid interface of this conjugate problems, case 1 and case 2, are given by equations (27) and (28), respectively. Furthermore, it is assumed that the axial heat conduction in the flat plate can be neglected.

Table 1. Example of calculated values of a_n

Case	1	1	2	2
K	1000	500	2000	598
D	100	50	50	150
Grl, Grl^*	10^6	10^6	8.1×10^9	10^6
a_0	0.9803	0.9827	5.6619	0.9600
a_1	0.0194	0.0097	0.0968	0.0158
a_2	−0.0242	−0.0112	0.0283	−0.0182
a_3	0.0198	0.0092	−0.0528	0.0150
a_4	0.0078	−0.0038	0.0009	0.0062

Then, heat balance at each axial station of the heated flat plate gives the following dimensionless interfacial temperature:

case 1:

$$F = \frac{1}{1 + A(Pr) \cdot (D/K) \cdot F^{1/4} X^{-1/4}} \tag{29}$$

case 2:

$$F = \frac{X^{1/5}}{B(Pr)} \tag{30}$$

The right hand side of equation (29) includes the parameter (D/K), which is the dominant parameter in equation (17), too. The interfacial temperature given by equation (30) is the same one which is given by the solution of constant interfacial heat flux.

5. NUMERICAL RESULTS

Numerical results were obtained for $Pr = 0.70$ alone and variations k_s and d. The present analytical method may be adapted to various Prandtl numbers with the corresponding local similarity solution of the boundary layer equations.

Table 1 shows examples of the calculated values of a_n. It seems that equation (21) is a rapidly convergent series. The order of the polynomial selected had no apparent effect on the solution for polynomials of third and fourth order polynomials.

Figures 2 and 3 show the comparisons of the interfacial temperature distributions, between the present conjugate solutions and finite-difference solutions, for case 1 and case 2. It can be seen that the difference between both results are insignificant except for the region near the leading edge of case 2. Present analytical conjugate solutions give about 6% higher interfacial temperature than the finite-difference solution at the leading edge, for case 2. Near the leading edge, a difficulty incapable of solution by the present treatment occurs. It is that the boundary layer approximations in equations (2) and (3) are not available near the leading edge. And it is improbable that there is a perfectly adiabatic flat plate under the heated plate. These problems will have to be investigated in the future.

Now, Fig. 4 shows the dimensionless interfacial temperature in case 1 for $Grl = 10^9$. In this figure, there are three groups of interfacial temperature corresponding to the three values of KL/D: 1000, 500 and 100. In each group, the interfacial temperatures have the common KL/D and the different K and D/L, as is shown in the figure. These results indicate that the controlled parameter of case 1 is KL/D, which appears also in the leading term of equation (17).

If the small effects of the boundary condition at the upper surface of cross section of the heated plate on the interfacial temperature are neglected, dimensionless length $L(= Grl^{1/3})$ in KL/D and X/L is insignificant; and the interfacial temperatures for Grl less than 10^9 are given by the same interfacial temperature for $Grl = 10^9$ with the same K/D and the same X $(= Grx^{1/3})$. The results of no axial conduction in the flat plate (equation (29)), indicated by the dotted chain lines in Fig. 4, give the interfacial temperature with little difference from the present conjugate solution. It can be seen that the effects of the axial conduction in the flat plate are insignificant in case 1.

Figure 5 shows the dimensionless interfacial temperature in case 2 for $Grl^* = 10^{10}$ $(L = Grl^{*1/4})$. It can be seen that with case 2 the controlled dimensionless parameter is KD by the equivalent considerations to case 1. But the simplified treatment of case 2 (equation

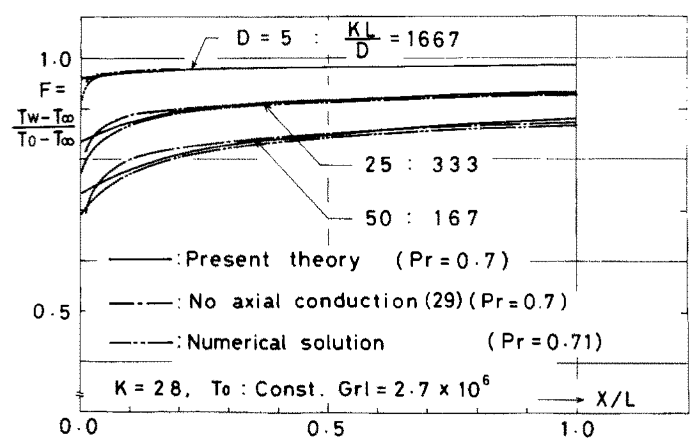

FIG. 2. Comparisons of interfacial temperature distributions, between present conjugate solutions and finite-difference solutions (case 1).

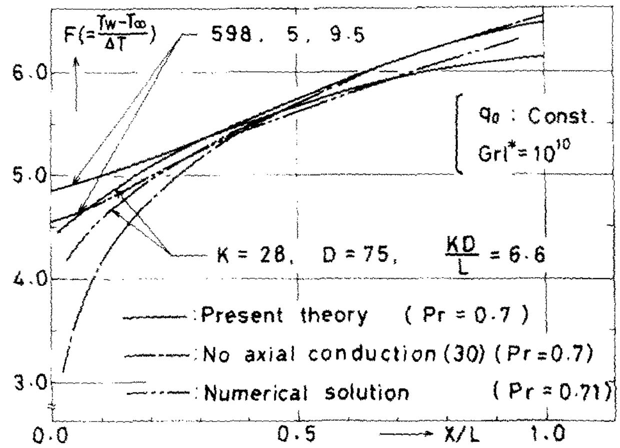

FIG. 3. Comparisons of interfacial temperature distributions, between present conjugate solutions and finite-difference solutions (case 2).

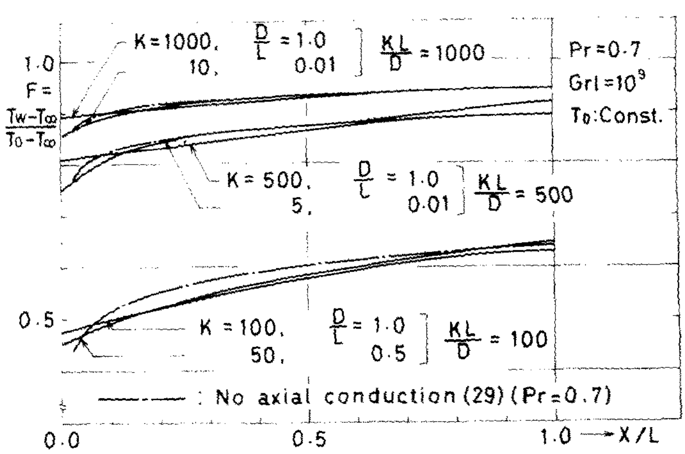

FIG. 4. Effect of dimensionless parameter KL/D on interfacial temperature profiles (case 1).

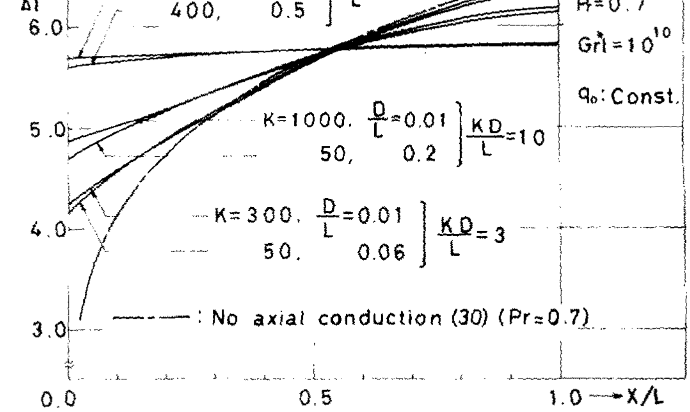

FIG. 5. Effect of dimensionless parameter KD/L on interfacial temperature profiles (case 2).

Table 2. Materials of the heated flat plate

Material	Thickness	Thermal conductivity	Emittance
Sus 304 Stainless steel	5 mm	$16\ W\ m^{-1}\ K^{-1}$	0.2
Aluminum	5 mm	$204\ W\ m^{-1}\ K^{-1}$	0.2
Glass	6 mm	$0.76\ W\ m^{-1}\ K^{-1}$	0.9

(30)), considering no axial conduction, gives the different interfacial temperature from the present conjugate solution. Larger values of KD cause significant axial conduction effects on the interfacial temperature; and the interfacial temperatures become nearly uniform for KD larger than 2.0×10^5.

Figure 6 shows the effect of dimensionless parameter KD on average Nusselt number of case 2 for $Grl^* = 10^{10}$. Average Nusselt numbers for the different K but the same KD are nearly equal, and larger values of KD give smaller Nusselt number being close to Nusselt number of the constant interfacial temperature.

6. EXPERIMENT

In order to compare the present analytical solutions with observation, experiments corresponding to case 2, were conducted on three materials; stainless steel, aluminum and glass (Table 2). A brief recapitulation of the experiments is in order here, and details of them are given in [12]. Heating stainless foils, 0.0015 mm in thickness, both surfaces of which were electrically insulated with polyester films, 0.1 mm in thickness, were sandwiched by the same two plates of the tested materials, as shown in Fig. 7. These piled up plates were 600 mm in width, 400 mm in height and about 11 mm in total thickness. The stainless foils of uniform thickness inserted between the tested plates were heated by putting an electric current through them and achieving the uniform surface flux condition. The heated flat plate was placed perpendicularly at the center in the veneer test chamber, which was 1815 mm in height, 925 mm in width and 925 mm in length, as is shown in Fig. 8. A veneer plate, 10 mm in thickness, was placed vertically under the heated plate, about 1 mm away from the leading edge of the heated plate, to minimize the heat loss from the leading edge.

In order to measure the surface temperature of the heated plate, thermocouples of copper constantan of 0.1 mm dia., were bonded on the surface of the heated plate. These thermocouple junctions were placed along the vertical center line of the heated plate and the individual wires were led across the heated plate, as is shown in Fig. 7. The spanwise temperature distributions of the heated plate was measured by the use of copper constantan thermocouples of 0.1 mm dia., which were embedded in the heated plate and were placed along two horizontal lines at different distances from the leading edge. Temperature stratifications in the test chamber and the temperature at the inner surface of the test chamber were measured by the use of copper constantan thermocouples of 0.35 mm dia., as is shown in Fig. 8. Thermal e.m.f. of the thermocouples was measured by the use of a digital voltmeter with a resolution of 0.001 mV.

Heat flux at the surface of the inserted stainless foils was determined by the previously measured resistance of the stainless foils and the measured current through the foils. In order to compare the experimental results of the surface temperature with the present conjugate solutions, the measured heat flux at the point, where the stainless foils were inserted, was corrected by thermal radiation and by a spanwise conduction heat loss in the heated plate. Heat flux qr of thermal radiation was evaluated by the following equation:

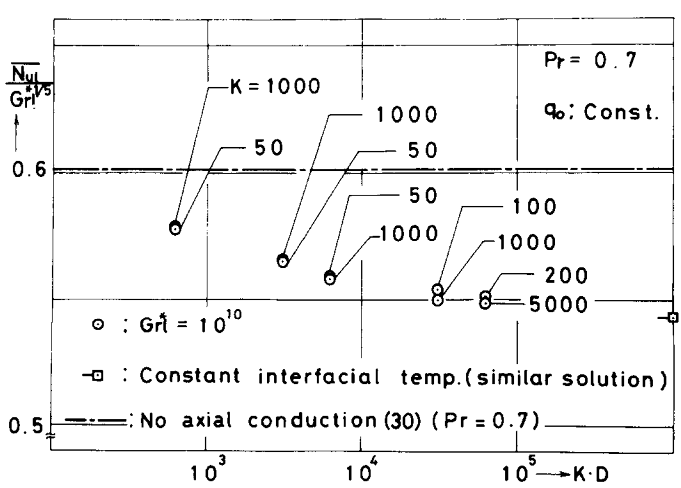

FIG. 6. Effect of dimensionless parameter KD on average Nusselt number (case 2).

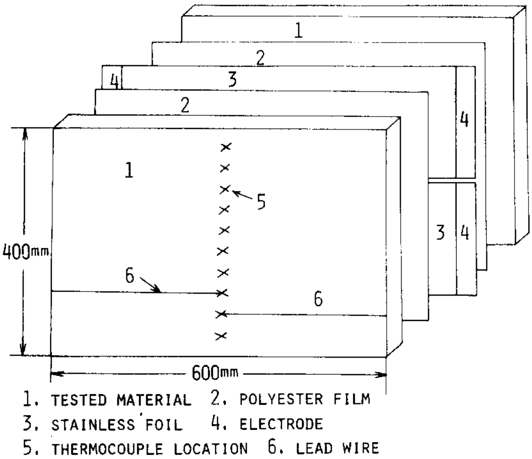

FIG. 7. Exploded view of the experimental heat transfer plate.

$$qr = \varepsilon\sigma(T_w^4 - T_v^4) \quad (31)$$

where σ is the Stefan–Boltzmann constant, ε is emittance from the surface of the heated plate, T_w is the surface temperature of the heated plate and T_v is the inner surface temperature of the test chamber. T_w and T_v are measured on the same horizon. The spanwise conduction heat loss in the heated plate was relatively small and can be roughly given by the following equation:

$$q\,\text{loss} = 2\,dk_s \frac{\partial^2 T_s}{\partial z^2} \quad (32)$$

Measured surface temperature T_w were transformed into the dimensionless surface temperature F by using q_0 in equation (10), which was determined by the following equation:

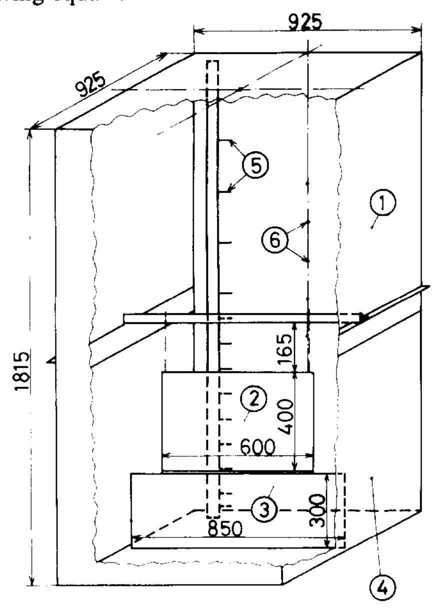

FIG. 8. Schematic diagram of the experimental apparatus: (1) veneer test chamber, (2) heated plate, (3) veneer plate, (4) acryl plate for visual flow, (5) thermocouples to measure temperature of surrounding fluid, (6) thermocouples to measure temperature of veneer plate.

$$q_0 = q - qr - q\,\text{loss} \quad (33)$$

where q was the measured heat flux at the point where the stainless foils were inserted. The physical properties of air were evaluated by the following reference temperature:

$$T_r = T_w - 0.38\,(T_w - T_\infty). \quad (34)$$

Figure 9 shows the comparisons, concerning dimensionless surface temperature profiles, between experimental results obtained by aforementioned corrections and the present conjugate solutions. The present conjugate solution agreed well with the experiment for a larger value of KD. When KD/L was 7.5 ($KD = 2372$), the difference between the present solution and the experiment was about 10% in the region near the leading edge, and might be introduced by both error in the experiment and error in present conjugate solution, as mentioned before.

7. CONCLUSION

The following can be concluded: Comparisons of present analytical solutions with finite-difference solutions and the experimental results indicate the practicality of the present conjugate solutions. Straightforward insight on this conjugate problem can be obtained by the use of the controlled dimensionless parameter K/D or KD.

1. When the outside surface of the flat plate is maintained at a uniform higher temperature, the controlled dimensionless parameter is K/D and axial heat conduction in the flat plate insignificantly affects the temperature distribution.

2. When the outside surface is heated at uniform heat flux, the controlled dimensionless parameter is KD and axial heat conduction in the flat plate has significant effects on the temperature distributions for larger KD. Larger values of KD than 10^5 give nearly uniform interfacial temperature.

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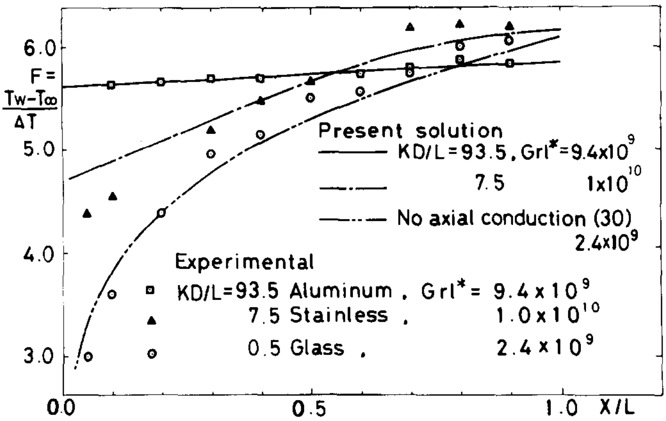

FIG. 9. Comparisons of interfacial temperature profiles, between present conjugate solutions and experimental results.

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EFFET DE LA CONDUCTION THERMIQUE AXIALE DANS UNE PAROI VERTICALE SUR LA CONVECTION NATURELLE THERMIQUE

Résumé—On analyse des problèmes de transferts thermiques couplés bidimensionnels et permanents, pour la convection naturelle sur une plaque verticale, par la méthode de similarité locale de la couche limite. Les deux conditions aux limites thermiques considérées ici sont de température constante et de flux constant sur la face externe de la plaque. Les effets de la conduction axiale dans la plaque sur la température interfaciale sont significatifs dans le cas du flux constant et ils sont unifiés par le paramètre sans dimension *KD*.

Des comparaisons avec les résultats de la méthode aux différences finies et avec les expériences montrent la validité des solutions analytiques présentées.

EINFLÜSSE DER AXIALEN WÄRMELEITUNG IN EINER VERTIKALEN EBENEN PLATTE AUF DEN WÄRMEÜBERGANG BEI FREIER KONVEKTION

Zusammenfassung — Das stationäre zweidimensionale gekoppelte Wärmeübergangsproblem bei freier Konvektion an einer vertikalen ebenen Platte wurde nach der Methode der Verwendung lokaler Ähnlichkeitslösungen der freien Konvektions-Grenzschicht untersucht. Die zwei in dieser Arbeit betrachteten thermischen Randbedingungen sind konstante Temperatur und konstanter Wärmestrom an der äußeren Oberfläche der ebenen Platte. Die Einflüsse der axialen Wärmeleitung in der Platte auf die Grenzschichttemperatur sind für den Fall konstanten Wärmestroms bedeutend und wurden durch den dimensionslosen Parameter *KD* korreliert. Vergleiche mit den Ergebnissen nach der Methode der finiten Differenzen und entsprechenden Experimenten bestätigten die Brauchbarkeit der vorliegenden analytischen Lösungen.

ВЛИЯНИЕ ПРОДОЛЬНОЙ ТЕПЛОПРОВОДНОСТИ ВЕРТИКАЛЬНОЙ ПЛОСКОЙ ПЛАСТИНЫ НА ТЕПЛОПЕРЕНОС ПРИ СВОБОДНОЙ КОНВЕКЦИИ

Аннотация — Используя метод, основанный на локальном автомодельном решении для пограничного слоя при свободной конвекции, проведен анализ сопряжённых задач стационарного двухмерного свободноконвективного теплопереноса к вертикальной плоской пластине при двух тепловых граничных условиях: постоянной температуре и постоянной плотности теплового потока на внешней поверхности плоской пластины. Продольная теплопроводность плоской пластины оказывает существенное влияние на распределение температур в слое при постоянном тепловом потоке и учитывается с помощью безразмерного параметра *KD*. Справедливость аналитических решений проверена путем сравнения с результатами, полученными методом конечных разностей, и с соответствующими экспериментальными данными.